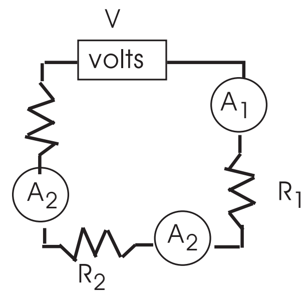
Chapter 12
Series and Parallel Circuits

Series Circuits



A **series circuit** is one in which there is only one current path. The current is the same in all the components of a series circuit. The sum of the potential drops in a series circuit is equal to the total applied potential difference. The equivalent resistance of the series circuit is equal to the sum of the resistance of its components and can be derived as follows.

	$\mathbf{V}$	=	$\mathbf{V_1 + V_2 + V_3 + \ldots}$
Where:	$\mathbf{V}$	=	$\mathbf{IR}$
Thus,	$\mathbf{IR_{eq}}$	=	$\mathbf{I_1 R_1 + I_2R_2 + I_3R_3 + \ldots}$
But,	$\mathbf{I}$	=	$\mathbf{I_1 = I_2 = I_3 = \ldots}$
So,	$\mathbf{R_{eq}}$	=	$\mathbf{R_1 + R_2 + R_3 = \ldots}$

$$\mathbf{I = I_1 = I_2 = I_3 = \ldots}$$

$$\mathbf{V = V_1 + V_2 + V_3 = \ldots}$$

$$\mathbf{R_{equivalent} = R_1 + R_2 + R_3 = \ldots}$$

Example

Three resistors of 20, 30, and 40 ohms are connected in series to an applied potential of 120 volts.

Calculate:

a total resistance
b current through each resistor
c potential drop across each resistor

Given: $R_1 = 20\ \Omega$ $R_3 = 40\ \Omega$
$R_2 = 30\ \Omega$ $V = 120\text{ V}$

Solution:

$$a \quad R_{eq} = R_1 + R_2 + R_3 = 20\ \Omega + 30\ \Omega + 40\ \Omega = 90\ \Omega$$

$$b \quad I = \frac{V}{R_{eq}} = \frac{120\ V}{90\ \Omega} = 1.3 \text{ amperes}$$

In the series circuit, current is the same in all parts of the circuit.

$$I = I_1 = I_2 = I_3$$

$$c \quad V_1 = I_1R_1 = (1.3\ A)(20\ \Omega) = 26\ V$$
$$V_2 = I_2R_2 = (1.3\ A)(30\ \Omega) = 39\ V$$
$$V_3 = I_3R_3 = (1.3\ A)(40\ \Omega) = 52\ V$$

In a series circuit, voltage total is equal to the sum of the voltage drops across each resistor.

Check:

$$V = V_1 + V_2 + V_3$$
$$= 26\ V + 39\ V + 52\ V = 117\ V$$
$$V = 120\ V$$

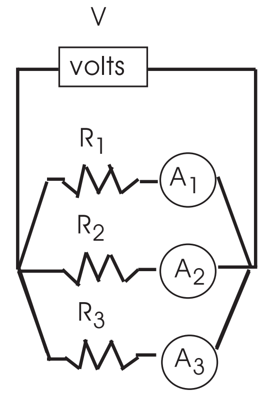
Real World Connections

The Tiny Life-saving Sensor

If too much current flows through a wire, it can get hot enough to set fire to surrounding materials. Fuses and circuit breakers protect against this by cutting off power to a circuit that is drawing excessive current. Circuit breakers work like thermostats, when they get hot they shut off. Fuses "blow" when a metal strip overheats and melts. The fuse and circuit breakers protect the wiring in the building. To protect a person against lethal shocks from leakage of current, a GFI outlet is installed. GFI, or ground fault interrupter, senses currents as low as 0.005 ampere. It senses when some of the current flowing out of one side of the outlet is not returning to the other side of the outlet. If the current isn't coming back, then the leakage could flow through your body to ground. The usual cause of leakage current in appliances or tools is a breakdown of insulation between the current carrying wire and the frame of the electrical device. Since water is such a good conductor, GFI outlets should be used in wet locations, such as the kitchen, bathroom, or outdoors.

Adapted from *Readers Digest Fix it Manua*l, 1977. Pg 127

PARALLEL CIRCUITS



A parallel circuit is one in which there is more than one current path. The potential drop is the same across each branch of a parallel circuit. The total current in a parallel circuit is equal to the sum of the branch currents. The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocals of the branch resistances and can be derived as follows:

$$I = I_1 + I_2 + I_3 + \ldots$$

Where: $I = V/R$

Thus, $$\frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \ldots$$

But, $V = V_1 = V_2 = V_3 = \ldots$

So, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$V = V_1 = V_2 = V_3 = \ldots$$

$$I = I_1 + I_2 + I_3 = \ldots$$

$$\frac{1}{R_{equiv.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Example: Three resistors of 20., 30., and 40. ohms are connected in parallel to an applied potential of 120 volts.

Calculate:
a the total resistance
b the potential difference across each resistor
c the current through each resistor

Given:
$R_1 = 20.\ \Omega$ $R_3 = 40.\ \Omega$
$R_2 = 30.\ \Omega$ $V = 120\ V$

Find:
a R_{total}
b V_1, V_2, V_3
c I_1, I_2, I_3

Solution:

a
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{20\Omega} + \frac{1}{30\Omega} + \frac{1}{40\Omega}$$

$$\frac{1}{R_{eq}} = \frac{6}{120\Omega} + \frac{4}{120\Omega} + \frac{3}{120\Omega}$$

$$\frac{1}{R_{eq}} = \frac{13}{120\Omega}\ , \ R_{eq} = \frac{120\Omega}{13} = 9.2\ \Omega$$

b In the parallel circuit, voltage is the same across each resistor in the circuit.

$$V = V_1 = V_2 = V_3 = 120\ V$$

c

$$I_1 = \frac{V_1}{R_1} = \frac{120\ \text{volts}}{20.\ \Omega} = 6.0\ \text{amp}$$

$$I_2 = \frac{V_2}{R_2} = \frac{120\ \text{volts}}{30.\ \Omega} = 4.0\ \text{amp}$$

$$I_3 = \frac{V_3}{R_3} = \frac{120\ \text{volts}}{40.\ \Omega} = 3.0\ \text{amp}$$

Note: In parallel circuits, the sum of the currents in the resistors is equal to the total current from the source

$$I = I_1 + I_2 + I_3$$

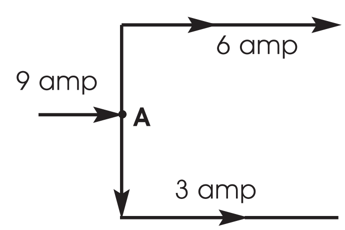
$$= 6.0\ A + 4.0\ A + 3.0\ A$$

$$I = 13\ A$$

Check: $$I = \frac{V}{R_{eq}} = \frac{120\ V}{9.2\ \Omega} = 13\ A$$

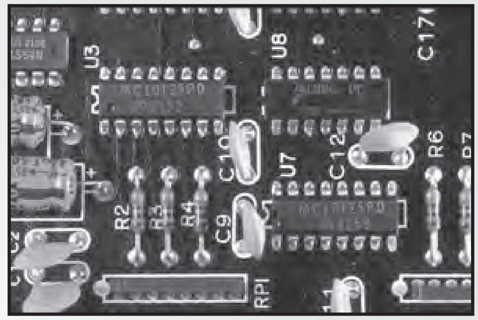
Kirchhoff's Laws

First Law – Conservation of Charge. The sum entering any current junction, is equal to the sum leaving. ***A*** represents a junction in an electric circuit. Nine amperes are entering ***A***; therefore, according to Kirchhoff's First Law, nine amperes must come out of junction ***A***.

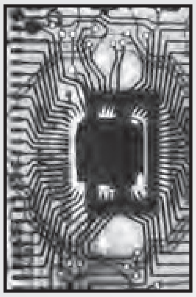


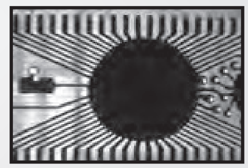
Real World Connections

Tiny Nanowire – Microcircuits



Millions of transistors, resistors, and conductors are assembled by a photographic process, in three dimensions. How small can you make a wire which still conducts? A 16 gauge copper wire is about 2 mm in diameter and can conduct 6 amps. In perspective, the human hair is 100 µm or 100,000 nm in diameter. Making micron size wires (1 µm or 1,000 nm) a factor of 100 or smaller than human hair involves the $300 billion semiconductor industry. The process of semiconductor lithography starts with a wafer of silicon. A series of circuit designs, depositions, ion implantations, and etching, which is all done through a stencil, forms a microcircuit capable of storing and retrieving millions of bits of information.





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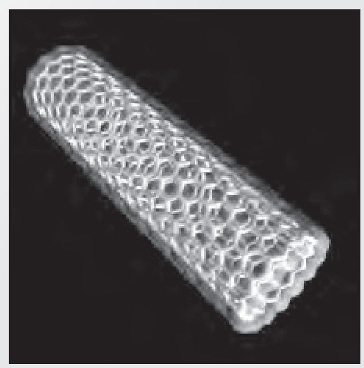
The microchip uses an electric circuit of micron wires made from aluminum or copper to connect the chip to a external circuit. Microminiaturization continues to drive circuits with wires as small as 100 nm to the limit of current capacity of copper – a limit of 1 million A/cm^2 (scales down with area). New materials made of single walled nanotubes of carbon, called Bucky tubes (1 nm in diameter), are being investigated to allow for better current carrying capacity for (billion A/cm^2) future circuits of 10 nm in size.

Source: *Scientific American*, Dec 2000 p.69



http://www.chem.ox.ac.uk/mom/buckytubes/buckytubes.html
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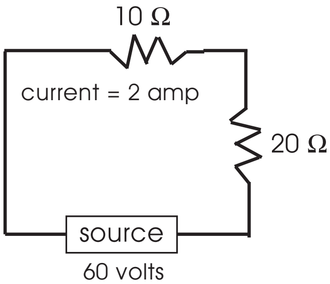


Second Law – Conservation of Energy. The algebraic sum of all the potential drops, and the applied voltage around a complete circuit is equal to zero. According to Kirchhoff's Second Law, the applied voltage (+60 volts) minus the potential drops around the complete circuit, must equal zero.

$$60 \text{ volts} - (20\ \Omega \times 2 \text{ amp}) - (10\ \Omega \times 2 \text{ amp}) = 0$$

$$60 \text{ volts} - 40 \text{ volts} - 20 \text{ volts} = 0$$

Skills 4.1xiii, 4.1viii, 4.1xii

Lab 20 – Series and Parallel Circuits

Background

The components of a circuit can be connected in series or in parallel. In a series circuit, the parts are connected end-to-end. When elements are connected in parallel, one bridges the other. An ammeter is always connected in series. A voltmeter, on the other hand, is always connected in parallel with the part of the circuit across which it is measuring a potential difference.

The Problem

The plan of this experiment is to study series and parallel circuits. In each case, you will measure the current in each component, the potential difference across each component, and the current and potential difference for the entire circuit. The final result for each part of the experiment is a comparison between the circuit resistance as computed from your current and voltage measurements and the circuit resistance computed from the known values of the individual resistors.

Preparing for the Experiment

Select three resistors or light bulbs of different resistances. The resistance of each must be known in advance. If the resistors are not labeled, measure the resistance of each by the voltmeter-ammeter method. Set up an appropriate data table to record all measurements. Compare your actual and computed results.

Series Circuit

Connect two of the resistors in series. Measure the current in each, the voltage across each, and the applied voltage. How are the separate voltages across the resistors related to the applied voltage? How is the current through the source related to the currents in the individual resistors? Calculate the circuit resistance from your measurements and compare it with the resistance computed from the known values of the resistors.

Parallel Circuit

Connect two of the resistors in parallel. Measure the current in each, the total circuit current, and the applied voltage. Again, state the relationship of the applied voltage and current to these quantities measured for the separate resistors. Calculate the combined resistance from your measurements and compare it with the resistance computed from the known resistances of the components.

☆ Try It

Base your answers to questions 1 through 3 on the information below.

> You are given a 12-volt battery, ammeter **A**, voltmeter **V**, resistor $\mathbf{R_1}$, and resistor $\mathbf{R_2}$. Resistor $\mathbf{R_2}$ has a value of 3.0 ohms.

1 Using appropriate symbols from the *Reference Tables for Physical Setting/Physics*, draw and label a complete circuit showing:

a resistors R_1 and R_2 connected in parallel with the battery [1]

b the ammeter connected to measure the current through resistor R_1, only [1]

c the voltmeter connected to measure the potential drop across resistor R_1 [1]

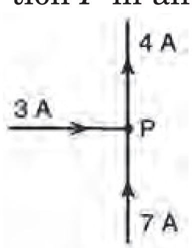
2 If the total current in the circuit is 6.0 amperes, determine the equivalent resistance of the circuit. [1]

3 If the total current in the circuit is 6.0 amperes, determine the resistance of resistor R_1. [Show all calculations, including the equation and substitution with units.] [2]

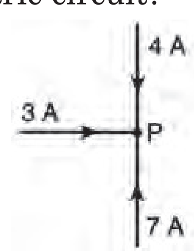
Chapter Twelve Assessments

Part A Questions

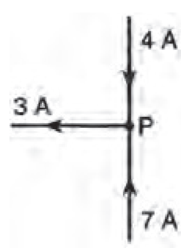
1 Which diagram below correctly shows currents traveling near junction *P* in an electric circuit?



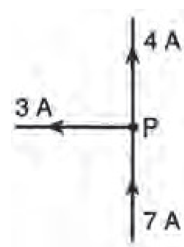
(1)



(2)



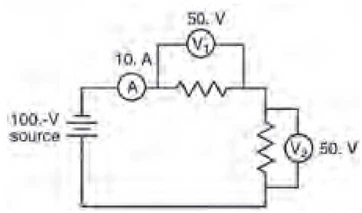
(3)



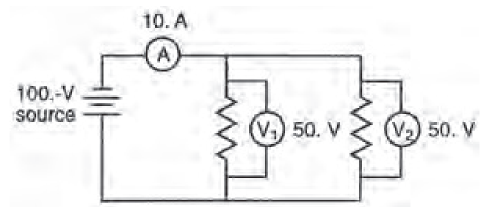
(4)

2 An electric circuit contains an operating heating element and a lit lamp. Which statement best explains why the lamp remains lit when the heating element is removed from the circuit?
(1) The lamp has less resistance than the heating element.
(2) The lamp has more resistance than the heating element.
(3) The lamp and the heating element were connected in series.
(4) The lamp and the heating element were connected in parallel.

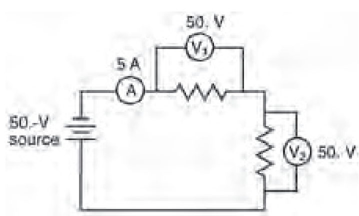
3 In which pair of circuits shown below could the readings of voltmeters $\mathbf{V_1}$ and $\mathbf{V_2}$ and ammeter **A** be correct?



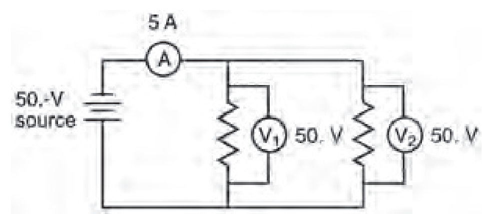
A



C



B

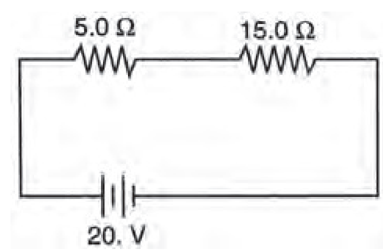


D

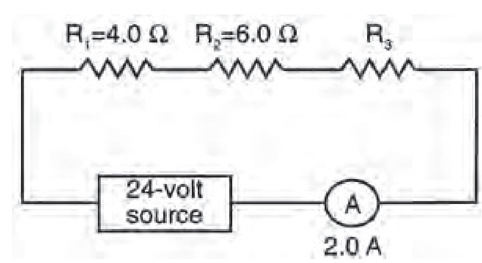
(1) *A* and *B* (2) *B* and *C* (3) *C* and *D* (4) *A* and *D*

Part B Questions

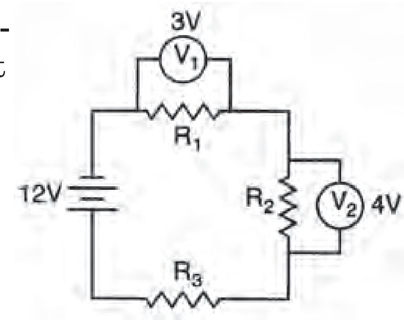
4 The diagram at the right shows two resistors connected in series to a 20.-volt battery. If the current through the 5.0-ohm resistor is 1.0 ampere, the current through the 15.0-ohm resistor is [1] ______ A



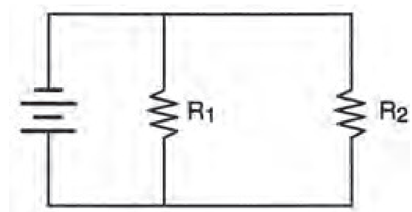
5 The diagram at the right shows a circuit with three resistors. What is the resistance of resistor R_3? [1] ______ Ω



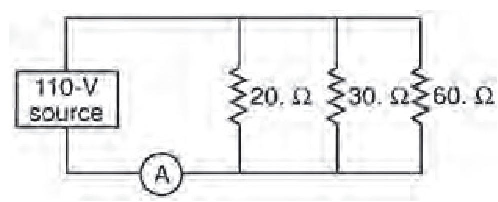
6 The diagram at the right shows three resistors, R_1, R_2, and R_3, connected to a 12-volt battery. If voltmeter V_1 reads 3 volts and voltmeter V_2 reads 4 volts, what is the potential drop across resistor R_3?
[1] ______ V



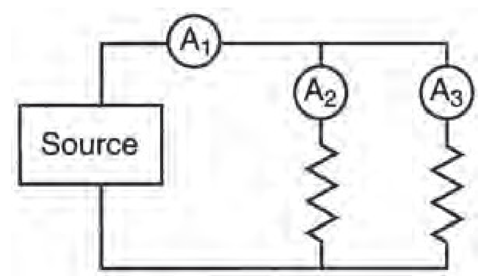
7 Identical resistors R_1 and R_2 have an equivalent resistance of 6 ohms when connected in the circuit shown at the right. The resistance of R_1 is [1]_____ Ω



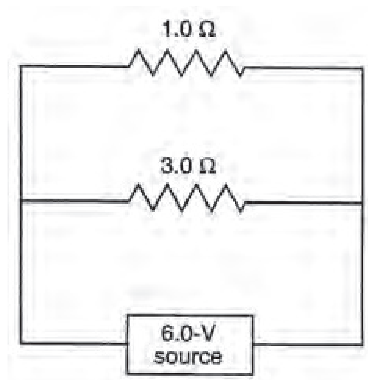
8 In the parallel circuit at the right, ammeter *A* measures the current supplied by the 110-volt source. The current measured by ammeter *A* is
[1] _____ *A*



9 Three ammeters are placed in a circuit as shown at the right. If A_1 reads 5.0 amperes and A_2 reads 2.0 amperes, what does A_3 read?
[1] _____ A



Base your answers to questions 10 and 11 on the diagram at the right, which shows two resistors connected in parallel across a 6.0-volt source.

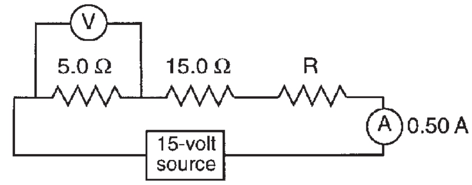


10 The equivalent resistance of the two resistors is [1] _______ Ω

11 Compared to the power dissipated in the 1.0-ohm resistor, the power dissipated in the 3.0-ohm resistor is [1] __________

PART C QUESTIONS

Base your answers to questions 12 and 13 on the information and diagram at the right.



A 5.0-ohm resistor, a 15.0-ohm resistor, and an unknown resistor, R, are connected as shown with a 15-volt source. The ammeter reads a current of 0.50 ampere.

12 Determine the reading of the voltmeter connected across the 5.0-ohm resistor. [Show all calculations, including the equation and substitution with units.] [2]

13 Determine the total electrical energy used in the circuit in 600. seconds. [Show all calculations, including the equation and substitution with units.] [2]

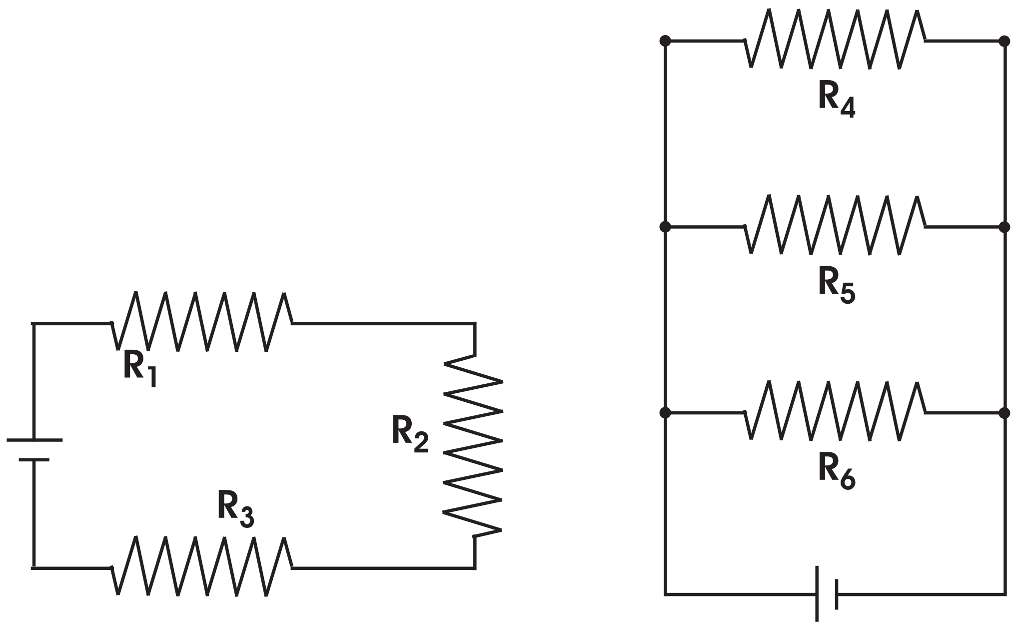
14 Two resistors are connected in parallel to a 12-volt battery. One resistor, R_1, has a value of 18 ohms. The other resistor, R_2, has a value of 9 ohms. The total current in the circuit is 2 amperes. A student wishes to measure the current through R_1, and the potential difference across R_2.

a Using the symbols from the *Reference Tables* for a battery, an ammeter, a voltmeter, and resistors, draw and label a circuit diagram that will enable the student to make the desired measurements. [2]

b Calculate the value of the current in resistor R_1. [Show all calculations, including equations and substitutions with units.] [2]

Question 15 on next page

15 Using the values given in the chart below and the diagrams of the circuits, complete the table. Determine the current through the resistor, the voltage across the resistor, and the power developed by the resistor. [3]



Series Circuit
Voltage Total = 195 v

Parallel Circuit
Voltage Total = 360 v

	Series			Parallel		
Resistor	R_1	R_2	R_3	R_4	R_5	R_6
Resistance Ω	13	15	11	45	180	36
Current *A*						
Voltage V						
Power W						